(https://www.cut-the-knot.org/manifesto/index.shtml)

Michael Rozenberg's Inequality in Two Variables

Problem

Prove that, if $x, y \ge 0,$ and $x+y=2,$ then

$$\sqrt{x^2+3}+\sqrt{y^2+3}+\sqrt{xy+3}\ge 6.$$

Solution 1

Solution 2

Illustration

Acknowledgment

Inequalities with the Sum of Variables as a Constraint

- *An Inequality for Grade 8 (https://www.cut-the-knot.org/arithmetic/algebra/AnInequality.shtml)* $\left(\frac{1-x_1}{1+x_1}\cdot\frac{1-x_2}{1+x_2}\cdot\ldots\cdot\frac{1-x_n}{1+x_n}\ge\frac{1}{3}\right)$
- *An Inequality with Constraint (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality.shtml)* $((x+1)(y+1)(z+1)\ge 4xyz)$
- *An Inequality with Constraints II (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality2.shtml)* $\left(abc+\frac{2}{ab+bc+ca}=p+\frac{2}{q}\ge q-2+\frac{2}{q}\right)$
- *An Inequality with Constraint V (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality5.shtml)* $\left(\prod_{k=1}^{n}x_k^{1/x_k}\le\frac{1}{n^{n^2}}\right)$
- *An Inequality with Constraint VI (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality6.shtml)* $\left(\prod_{k=1}^{n}\frac{1+x_k}{x_k}\ge\prod_{k=1}^{n}\frac{n-x_k}{1-x_k}\right)$
- *An Inequality with Constraint XI (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality11.shtml)* $(\sqrt{5a+4}+\sqrt{5b+4}+\sqrt{5c+4}\ge 7)$
- *Monthly Problem 11199 (https://www.cut-the-knot.org/arithmetic/algebra/MonthlyProblem11199.shtml)* $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ge\frac{25}{1+48abc}\right)$
- *Problem 11804 from the AMM (https://www.cut-the-knot.org/arithmetic/algebra/AMM_11804.shtml)* $(10|x^3+y^3+z^3-1|\le 9|x^5+y^5+z^5-1|)$
- *Sladjan Stankovik's Inequality With Constraint (https://www.cut-the-knot.org/arithmetic/algebra/Stankovnik.shtml)* $\left(abc+bcd+cda+dab-abcd\le\frac{27}{16}\right)$
- *Sladjan Stankovik's Inequality With Constraint II (https://www.cut-the-knot.org/arithmetic/algebra/Stankovik2.shtml)* $(a^4+b^4+c^4+d^2+4abcd\ge 8)$
- *An Inequality with Constraint V (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality5.shtml)* $\left(\prod_{k=1}^{n}x_k^{1/x_k}\le\frac{1}{n^{n^2}}\right)$
- *An Inequality with Constraint VI (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality6.shtml)* $\left(\prod_{k=1}^{n}\frac{1+x_k}{x_k}\ge\prod_{k=1}^{n}\frac{n-x_k}{1-x_k}\right)$
- *An Inequality with Constraint XII (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality12.shtml)* $(abcd\ge ab+bc+cd+da+ac+bd-5)$
- *An Inequality with Constraint XIII (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality13.shtml)* $((3a-bc)(3b-ca)(3c-ab)\le 8a^2b^2c^2)$

- *Inequalities with Constraint XV and XVI (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality15.shtml)* $\left(\frac{a^2}{\sqrt{b^2+4}}+\frac{b^2}{\sqrt{c^2+4}}+\frac{c^2}{\sqrt{a^2+4}}>\frac{3}{5}\right)$ and $\left(\frac{a^2}{\sqrt{b^4+4}}+\frac{b^2}{\sqrt{c^4+4}}+\frac{c^2}{\sqrt{a^4+4}}>\frac{3}{5}\right)$
- *An Inequality with Constraint XVII (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequality17.shtml)* $(a^3+b^3+c^3\ge 0)$
- *An Inequality with Constraint in Four Variables (https://www.cut-the-knot.org/m/Algebra/InequalityWithConstraintInFourVariables.shtml)* $\left(\frac{a^3}{b+c}+\frac{b^3}{c+d}+\frac{c^3}{d+a}+\frac{d^3}{a+b}\ge\frac{1}{8}\right)$
- *An Inequality with Constraint in Four Variables II (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequalityInFourVariables.shtml)* $(a^3+b^3+c^3+d^3+6abcd\ge 10)$
- *An Inequality with Constraint in Four Variables III (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequalityInFourVariables3.shtml)* $\left(abcd+\frac{15}{2(ab+ac+ad+bc+bd+cd)}\ge\frac{9}{a^2+b^2+c^2+d^2}\right)$
- *An Inequality with Constraint in Four Variables IV (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequalityInFourVariables4.shtml)* $\left(27+3(abc+bcd+cda+dab)\ge\sum_{cycl}a^3+54\sqrt{abcd}\right)$
- *Inequality with Constraint from Dan Sitaru's Math Phenomenon (https://www.cut-the-knot.org/arithmetic/algebra/SitaruMathPhenomenon.shtml)* $\left(b+2a+20\ge 2\sum_{cycl}\frac{a^2+ab+b^2}{a+b}\ge b+2c+20\right)$
- *An Inequality with a Parameter and a Constraint (https://www.cut-the-knot.org/m/Algebra/InequalityWithParameterAndConstraint.shtml)* $\left(a^4+b^4+c^4+\lambda abc\le\frac{\lambda+1}{27}\right)$
- *Cyclic Inequality with Square Roots And Absolute Values (https://www.cut-the-knot.org/m/Algebra/CyclicInequalityWithSquareRootsAndAbsoluteValues.shtml)* $\left(\prod_{cycl}\left(\sqrt{a-a^2}+\frac{1}{2\sqrt{2}}|3a-1|\right)\ge\frac{1}{6\sqrt{6}}\prod_{cycl}\left(\sqrt{a}+\frac{1}{\sqrt{3}}\right)\right)$
- *From Six Variables to Four - It's All the Same (https://www.cut-the-knot.org/m/Algebra/FromSixToFour.shtml)* $\left(\frac{5}{2}\le a^2+b^2+c^2+d^2\le 5\right)$

- *Michael Rozenberg's Inequality in Three Variables with Constraints (https://www.cut-the-knot.org/m/Algebra/MRozenbergInequalityWithConstraints.shtml)* $\left(4\sum_{cycl}ab(a^2+b^2)\ge\sum_{cycl}a^4+5\sum_{cycl}a^2b^2+2abc\sum_{cycl}a\right)$
- Michael Rozenberg's Inequality in Two Variables $\left(\sqrt{x^2+3}+\sqrt{y^2+3}+\sqrt{xy+3}\ge 6\right)$
- *Dan Sitaru's Cyclic Inequality in Three Variables II (https://www.cut-the-knot.org/arithmetic/algebra/DanSitaruCyclicInequality2.shtml)* $\left(\sum_{cycl}\sqrt{1+\frac{1}{a^2}+\frac{1}{(a+1)^2}}\ge\frac{9}{12-2(ab+bc+ca)}+3\right)$
- *Dan Sitaru's Cyclic Inequality in Three Variables IV (https://www.cut-the-knot.org/arithmetic/algebra/DanSitaruCyclicInequality4.shtml)* $\left(\sum_{cycl}\frac{(x+y)z}{\sqrt{4x^2+xy+4y^2}}\le 2\right)$
- *Dan Sitaru's Cyclic Inequality in Three Variables VI (https://www.cut-the-knot.org/arithmetic/algebra/DanSitaruCyclicInequality6.shtml)* $\left(\sum_{cycl}\left[\sqrt{a(a+2b)}+\sqrt{b(b+2a)}\right]\le 6\sqrt{3}\right)$
- *An Inequality with Arbitrary Roots (https://www.cut-the-knot.org/m/Algebra/InequalityWithArbitraryRoots.shtml)* $\left(\sum_{cycl}\left(\sqrt[n]{a+\sqrt[n]{a}}+\sqrt[n]{a-\sqrt[n]{a}}\right)<18\right)$
- *Inequality 101 from the Cyclic Inequalities Marathon (https://www.cut-the-knot.org/m/Algebra/Marathon101.shtml)* $\left(\sum_{cycl}\frac{c^5}{(a+1)(b+1)}\ge\frac{1}{144}\right)$
- *Sladjan Stankovik's Inequality With Constraint II (https://www.cut-the-knot.org/arithmetic/algebra/Stankovik2.shtml)* $(a^4+b^4+c^4+d^2+4abcd\ge 8)$
- *An Inequality with Constraint in Four Variables (https://www.cut-the-knot.org/m/Algebra/InequalityWithConstraintInFourVariables.shtml)* $\left(\frac{a^3}{b+c}+\frac{b^3}{c+d}+\frac{c^3}{d+a}+\frac{d^3}{a+b}\ge\frac{1}{8}\right)$
- *An Inequality with Constraint in Four Variables IV (https://www.cut-the-knot.org/arithmetic/algebra/ConstraintInequalityInFourVariables4.shtml)* $\left(27+3(abc+bcd+cda+dab)\ge\sum_{cycl}a^3+54\sqrt{abcd}\right)$
- *Cyclic Inequality with Square Roots And Absolute Values (https://www.cut-the-knot.org/m/Algebra/CyclicInequalityWithSquareRootsAndAbsoluteValues.shtml)*

$\left(\prod_{cycl}\left(\sqrt{a-a^2}+\frac{1}{2\sqrt{2}}|3a-1|\right)\geq\frac{1}{6\sqrt{6}}\prod_{cycl}\left(\sqrt{a}+\frac{1}{\sqrt{3}}\right)\right)$

- *From Six Variables to Four - It's All the Same (https://www.cut-the-knot.org/m/Algebra/FromSixToFour.shtml)* $\left(\frac{5}{2}\leq a^2+b^2+c^2+d^2\leq 5\right)$
- Michael Rozenberg's Inequality in Two Variables $\left(\sqrt{x^2+3}+\sqrt{y^2+3}+\sqrt{xy+3}\geq 6\right)$
- *Dan Sitaru's Cyclic Inequality in Three Variables II (https://www.cut-the-knot.org/arithmetic/algebra/DanSitaruCyclicInequality2.shtml)* $\left(\sum_{cycl}\sqrt{1+\frac{1}{a^2}+\frac{1}{(a+1)^2}}\geq\frac{9}{12-2(ab+bc+ca)}+3\right)$
- *Dorin Marghidanu's Two-Sided Inequality (https://www.cut-the-knot.org/m/Algebra/TwoSidedMarghidanu.shtml)* $\left(64(a+bc)(b+ca)(c+ab)\leq 8(1-a^2)(1-b^2)(1-c^2)\leq(1+a)^2(1+b)^2(1+c^2)\right)$
- *Problem 6 from Dan Sitaru's Algebraic Phenomenon (https://www.cut-the-knot.org/arithmetic/algebra/AlgebraicPhenomenon6.shtml)* $\left(x\sqrt{y+1}+y\sqrt{z+1}+z\sqrt{x+1}\leq 2\sqrt{3}\right)$
- *A Warmup Inequality from Vasile Cirtoaje (https://www.cut-the-knot.org/m/Algebra/WarmupFromVasileCirtoaje.shtml)* $\left(a^4b^4+b^4c^4+c^4a^4\leq 3\right)$
- *An Extension of the AM-GM Inequality (https://www.cut-the-knot.org/arithmetic/algebra/AM-GMExtension.shtml)* $\left(x_1x_2+x_2x_3+x_3x_4+\ldots+x_{99}x_{100}\leq\frac{1}{4}\right)$
  - *An Extension of the AM-GM Inequality: A second look (https://www.cut-the-knot.org/proofs/AM-GMExtensionSecondLook.shtml)* $\left(x_1x_2+x_2x_3+x_3x_4+\ldots+x_{n-1}x_n\leq\frac{a^2}{4}\right)$
- *Distance Inequality (https://www.cut-the-knot.org/m/Algebra/DistanceInequality.shtml)* $\left(a^2+b^2+c^2\leq\frac{9}{2}\right)$
- *Kunihiko Chikaya's Inequality with a Constraint (https://www.cut-the-knot.org/m/Algebra/KunnyWithConstraint.shtml)* $\left(4a^3+9b^3+36c^3\geq 1\right)$ *An Inequality with Five Variables, Only Three Cyclic (https://www.cut-the-knot.org/m/Algebra/FiveVariablesThreeCyclic.shtml)* $\left(\left(a+\frac{b}{c}\right)^4+\left(a+\frac{b}{d}\right)^4+\left(a+\frac{b}{e}\right)^4\geq 3(a+3b)^4\right)$
- *Second Pair of Twin Inequalities: Twin 1 (https://www.cut-the-knot.org/m/Algebra/twin21.shtml)* $\left(\prod_{i=1}^{n}\left(\frac{1}{a_i^2}-1\right)\geq(n^2-1)^n\right)$
- *Second Pair of Twin Inequalities: Twin 2 (https://www.cut-the-knot.org/m/Algebra/twin22.shtml)* $\left(\prod_{i=1}^{n}\left(\frac{1}{a_i}+1\right)\geq(n+1)^n\right)$

- *Cyclic Inequality In Three Variables from the 2018 Romanian Olympiad, Grade 9 (https://www.cut-the-knot.org/m/Algebra/CyclicInequalityFrom9GradeRomanianOlympiad.shtml)* $\left(\frac{a-1}{b+1}+\frac{b-1}{c+1}+\frac{c-1}{a+1}\ge 0\right)$
- *Dan Sitaru's Cyclic Inequality in Three Variables IX (https://www.cut-the-knot.org/arithmetic/algebra/DanSitaruCyclicInequality9.shtml)* $\left(\sum_{cycl}\sqrt{(x+y+1)(y+z+1)}\le 6+\sum_{cycl}\frac{x^3+y^3}{x^2+y^2}\right)$
- *Vasile Cirtoaje's Cyclic Inequality with Three Variables (https://www.cut-the-knot.org/m/Algebra/CyclicCirtoaje.shtml)* $\left(\sqrt{\frac{a}{b+c}}+\sqrt{\frac{b}{c+a}}+\sqrt{\frac{c}{a+b}}\ge 2\right)$ *Leo Giugiuc and Vasile Cirtoaje's Cyclic Inequality (https://www.cut-the-knot.org/m/Algebra/CyclicGiugiucCirtoaje.shtml)* $\left(\sqrt{\frac{a}{1-a}}+\sqrt{\frac{b}{1-b}}+\sqrt{\frac{c}{1-c}}+\sqrt{\frac{d}{1-d}}\ge 2\right)$

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